EVALUACIÓN DE MODELOS DE SIMULACIÓN DE CALIDAD DE AGUAS MEDIANTE TÉCNICAS DE ANÁLISIS GLOBAL DE SENSIBILIDAD E INCERTIDUMBRE: APLICACIÓN AL MODELO DE FILTROS VEGETALES VFSMOD-W

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ABSTRACT. This study proposes a two step model evaluation framework using global techniques: 1) a screening method (Morris method) for qualitative ranking of parameters and 2) a variance-based method (Extended Fourier Analysis Sensitivity Test – Extended FAST) for quantitative sensitivity and uncertainty analyses.

The techniques are applied to VFSMOD-W, a model used for designing vegetative filter strips. Regional characteristics of phosphate-mining region of central Florida along the Peace River basin, were used to construct the probability distributions of input factors. Two design filter lengths (3 and 6 m) and two alternative model structures were evaluated. The results clearly illustrate four possible products of the global sensitivity analysis: evaluation of the model’s behavior, ranking of importance of the parameters for different outputs, effect of changing modeling structure, and type of influence of the important parameters (first order or interactions).

RESUMEN. Este estudio propone un marco para la evaluación de la sensibilidad basado en el uso de dos técnicas globales: un método cualitativo y rápido para prospección (Método de Morris), y un método cuantitativo basado en el análisis de varianza de la salida del modelo (FAST, Test de Análisis de Sensibilidad de Fourier). El método rápido permite la identificación de un número reducido de los parámetros de entrada en base a su importancia relativa (sensibilidad del modelo) para el posterior análisis de sensibilidad e incertidumbre del modelo mediante FAST. Se presentan algunas aplicaciones de estas técnicas al modelo (VFSMOD-W) usado como herramienta para el diseño de filtros vegetales. Los valores de campo medidos en la zona y otras características locales conocidas fueron utilizados para la construcción de funciones de probabilidad para todas las entradas inciertas del modelo. Se evaluaron dos longitudes de diseño para filtros vegetales propuestas para la zona (3 y 6 m) y dos estructuras alternativas del modelo VFSMOD-W. Los resultados ilustran claramente cuatro de los productos del análisis global: control de calidad del modelo, importancia relativa de los parámetros de entrada en base a la sensibilidad del modelo, efecto del cambio de estructura del modelo, y tipo e influencia de los parámetros importantes.

1.- Introduction

Water quality models provide an alternative to field monitoring that potentially can save time, reduce cost, and minimize the need for testing management alternatives; however, the uncertainty of the model results is often a major concern (Shirmohammadi et al., 2006). Mathematical models are built in the presence of uncertainties of various types (parameter input variability, model algorithms or structure, model calibration data, scale, model boundary conditions, etc.) (Haan, 1989; Beven, 1989; Luis and McLaughlin, 1992). In a broad sense all these sources of uncertainty that can affect the variability of the model output have been referred to as “input factors”. The role of the sensitivity analysis is to determine the strength of the relation between a given uncertain input factor and the model outputs. The role of the uncertainty analysis is to propagate uncertainties in input factors onto the model outputs of interest (Saltelli et al., 2004). Since hydrological and water quality models are often complex and contain a large number of input factors, the evaluation of model sensitivity and uncertainty must be an essential part of the modeling process (Haan, 1989, 2002; Reckhow, 1994; Beven, 2006). If model uncertainty is not evaluated formally, the science and value of the model will be undermined (Beven, 2006). The issue of uncertainty of model outputs has policy, regulatory, and management implications, but understanding the source and magnitude of uncertainty and its effect on water-quality assessment has not been studied comprehensively (Beven, 2006; Muñoz-Carpena et al., 2006; Shirmohammadi et al., 2006).

The formal application of sensitivity and uncertainty analyses allows the modeler to: a) examine model behaviour; b) simplify the model; c) identify important input factors and interactions to guide the calibration of the model; d) identify input data or parameters that should be measured or estimated more accurately to reduce the uncertainty of the model outputs; e) identify optimal locations where additional data should be measured to reduce the uncertainty of the model; and f) quantify the uncertainty of the modeling results (Saltelli et al. 2005). However, in spite of advantages, these analyses are frequently ignored in water quality modeling efforts today (Beven, 2006; Shirmohammadi et al., 2006).

Traditionally, model sensitivity has been expressed mathematically as derivatives of the model output with
respect to the input variation, sometimes normalized by either the central values where the derivative is calculated or by the standard deviations of the input and output values (Haan, 1995). These sensitivity measurements are "local" because they are fixed to a point (base value) or narrow range where the derivative is taken. These local sensitivity indexes, used in "one-parameter-at-a-time" (OAT) methods, quantify the effect of a single parameter $X_i$ by assuming all others are fixed (Saltelli et al., 2005). Local sensitivities are used widely in hydrologic modeling (Haan, 1995) and are the basis of many applications, such as the solution of inverse problems (Cacuci, 2003). However, they are only fully informative if all factors in a model produce linear output responses, or if some sort of average can be used over the parametric space. Often models are non-linear and alternative "global" sensitivity approaches, where the entire parametric space of the model is explored simultaneously for all input factors, is more appropriate.

In addition, most global techniques, unlike OAT methods, provide information not only about the direct (first order) effect of the individual factors over the output, but also about their interaction (higher order) effects. Different types of global sensitivity methods can be selected based of the objective of the analysis (Saltelli et al., 2000, 2004, 2005).

Recently, Saltelli et al. (2004, 2005) proposed that a desirable statistical framework for model evaluation should be based on a set of global analyses techniques that meet the following requirements: a) are model-independent so they can be used with any model without modification; b) contain a screening method to efficiently identify the subset of important inputs controlling the output variability; c) contain a method that based on the reduced set of sensitive inputs can provide a quantitative decomposition of the output variance in term not only of first order but also higher order effects of the input factors; and d) allows for uncertainty analysis of the model based on the construction of PDFs using outputs derived from the variance-based method.

A framework meeting the above objectives is applied for VFSMOD-W in two steps: 1) screening by the method of Morris (1991) is applied. This method provides, with a comparatively small number of simulations, a qualitative ranking of input factors in terms of their relative effect over the model output; 2), if quantitative sensitivity information is desired, a variance-based technique (like Extended FAST) is performed at the expense of a larger number of simulations. The above methodology can be applied to a wide spectrum of models and applications and is especially efficient for computationally expensive models, or if a large number of parameters need to be evaluated simultaneously.

The consideration of model sensitivity and uncertainty should be linked to the availability or efficient collection of data. Ideally, uncertainty is quantified by probability distribution functions (PDFs) of the model outputs (Haan, 1989, 2002; Haan et al., 1995; Shirmohammadi et al., 2006). These PDFs can be used for decision by placing confidence levels on the outputs, usually in the form of a margin of safety (MOS) component, or by calculating a probability of exceedance of a maximum allowed value. An extensive review of uncertainty analysis methods applied to environmental models can be found in Morgan and Henrion (1992), Haan (2002) and Shirmohammadi et al., (2006).

2.- Sensitivity and Uncertainty methods and Application

2.1. Global Sensitivity and Uncertainty Analyses Techniques: Morris and Extended FAST

2.1.1. Screening Method: The Method of Morris

Parameter-screening methods (Saltelli et al. 2005) are designed to determine, in terms of model output, which of the model factors can be considered (1) negligible, (2) linear and additive, or (3) either non-linear or involved in interactions with other parameters. The screening method proposed by Morris (1991), (hereafter "Morris method" or "Morris") and later modified by Campolongo et al. (2005), was used here because it is relatively easy to implement, requires very few simulations, and interpreting its results is straightforward (Saltelli et al. 2005).

Morris (1991) proposed conducting individual randomized experiments that evaluate the effects of changing one parameter at a time. Each input may assume a discrete set of equispaced values, called levels, from an allocated range of variation for the factor. The elementary effect (di(x), local derivative of output in respect to input) for factor $X_i$ is defined as:

$$d_i (x) = \frac{[y(x_1,...,x_{i-1},x_i + \Delta, x_{i+1},x_k) – y(x)]}{\Delta} \quad (1)$$

where $x_i + \Delta$ - perturbed value of $x_i$, $i = 1,..., k$; $k$ – number of factors,

The principle of Morris method is to calculate the elementary effects, for values sampled at each level of factor $X_i$. The resulting elementary effects of factor $X_i$ are characterized by their mean and standard deviation.

For each parameter, two sensitivity measures are proposed by Morris (1991): (1) the mean of the elementary effects, $\mu$, which estimates the overall effect of the parameter on a given output; and (2) the standard deviation of the effects, $\sigma$, which estimates the higher-order characteristics of the parameter (such as curvatures and interactions). Since sometimes the model output is non-monotonic, Campolongo et al. (2005) suggested considering the distribution of absolute values of the elementary effects, $\mu^*$, to avoid the canceling of effects of opposing signs, and thus, $\mu^*$ and $\sigma$ were adopted as global sensitivity indexes in this method. The number of simulations ($N$) to perform in the analysis results as:

$$N = r (k + 1) \quad (2)$$

where $r$ - sampling size $r$ for search trajectory ($r = 10$)

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produces satisfactory results), $k$ - number of factors.

Although elementary effects are local measures, the method, is considered global, as the final measure $\mu^*$ is obtained by averaging the elementary effects which eliminates the need to consider the specific points at which they are computed (Saltelli et al., 2005).

To interpret the results in a manner that simultaneously accounts for the mean and standard deviation sensitivity measures, Morris (1991) suggested plotting the points on a $\mu$- $\sigma$ Cartesian plane. Morris (1991) recommended applying $\mu$ (or $\mu^*$ thereof) to rank parameters in order of importance. Saltelli et al. (2004) suggested applying the original Morris (1991) measure, $\sigma$, when examining the effects due to interactions. The meaning of $\sigma$ can be interpreted as follows: if its value is high for a parameter, $X_i$, the elementary effects relative to this parameter are sensitive to the chosen values of other parameters that constitute the remainder of the input space. Because the Morris method is qualitative in nature, it should only be used to assess the relative parameter ranking

### 2.1.2. Variance-based Method: Extended Fourier Amplitude Sensitivity Test (FAST) and Uncertainty Analysis

When a quantitative measure of sensitivity is to be obtained a variance-based method like the Fourier Amplitude Sensitivity Test (FAST) can be used (Cukier et al., 1973, 1978; Koda et al. 1979). Cukier et al. (1978) proposed that for independent factors, the total output variance can be expressed as:

$$V(Y) = \sum_i V_i + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk} + \ldots + V_{123\ldots k} \quad (3)$$

where multiple combinations of subindices (ij, ijl, ...,123...k) represent interactions of the factors.

Although FAST was originally developed to estimate the first-order effects of orthogonal inputs on a given model output, it has been extended to incorporate calculation of the total-order effects by Saltelli et al. (1999). FAST decomposes the total variance ($V=\sigma^2 Y^2$) of the model output, $Y=f(X_1, X_2, \ldots, X_i)$, using spectral analysis so that:

$$V = V_1 + V_2 + V_3 + \ldots + V_k + R \quad (4)$$

where $V_i$ is the part of the variance that can be attributed to the input factor $X_i$ alone, $k$ is the number of uncertain factors, and $R$ is a residual. The fraction of the total output variance attributed to a single factor can then be taken as a measure of global sensitivity of $Y$ with respect to $X_i$, i.e. the first order sensitivity index $S_i$, as

$$S_i = V_i / V \quad (5)$$

It is standard practice to assume that all parameters are uniformly distributed in [0,1] (Saltelli et al. 2004 and 2005), thereby permitting all parameters to be mapped from the unit hypercube to their actual distribution. To calculate $S_i$, the FAST technique randomly samples the $k$-dimensional space of the input parameters using the replicated Latin hypercube sampling (r-LHS) design (McKay et al., 1979, McKay 1995). The number of evaluations required in the analysis can be expressed as,

$$N = M (k + 2) \quad (6)$$

where $M$ is a number between 500-1000.

Higher-order interaction terms in equation (3) correspond to the residual $R$ contained in equation (4). Therefore, the sum of all $S_i$ is the fraction of total variance attributed to the sum of all the first-order effects. For a perfectly additive model, $\Sigma S_i = 1$; that is, no interactions are present and total output variance is explained as a summation of the individual variances introduced by varying each parameter alone. In general, models are not perfectly additive and $\Sigma S_i < 1$.

Extended FAST (Saltelli et al., 1999) allows for the determination of the higher order terms, which indicate the degree of parameter interaction. Another index, $ST_i$, (total sensitivity index for $X_i$) is calculated as the sum of the first order index and all higher order interaction-indices of a given parameter. For example, for parameter number 1:

$$ST_1 = S_1 + S_{1i} + S_{1jk} + \ldots + S_{1\ldots n} \quad (7)$$

For a given parameter, $X_i$, interactions can be isolated by calculating $ST_i - S_i$, which makes the extended FAST a powerful method for quantifying the individual effect of each parameter alone ($S_i$) or through interaction with others ($ST_i - S_i$). If individual quantification of the higher order interaction groups is desired Saltelli (2004) proposes the use of the method of Sobol (1990), although, since it is based on Monte-Carlo sampling, it typically requires a larger number of simulations than the Extended FAST.

An additional benefit of the Extended FAST analysis is that the results can be used for the uncertainty evaluation by constructing cumulative probability functions (CDFs) for each of the selected outputs.

It should be noted that the results of any model evaluation are specific to the particular application of the model. A “worst case scenario” where all the potentially sensitive model parameters are allowed to vary across their total (potential) parametric space could be implemented, in particular applications. It is important, however, that the user restricts the potential variation range or fixes some parameters, based on local field data or other information available. This practice can substantially change the uncertainty predictions, especially if the model is sensitive to the parameters that are fixed or have reduced range.
2.2. Analysis Procedure

In general, the proposed analysis procedures follow six main steps (Fig. 1): (1) selection of input factors and construction of PDFs; (2) generation of input sets by pseudo-random sampling of input PDFs according to the selected analysis method; (3) model runs for each input set; (4) global sensitivity analysis according to the method selected; (5) if the Morris screening method is selected, a subset of important parameters is obtained as a result, and steps 2-4 are repeated using the variance based methods; (6) uncertainty is assessed based on the outputs from the randomized variance-based model results by constructing PDF and the results are communicated to the end-users.

Fig. 1. General schematic for the global sensitivity and uncertainty analysis of models. Numbers in circles represent the steps in the global evaluation procedure explained in the text.

The software package, SimLab v2.2 (Saltelli et al., 2004), was used in the VFSMOD-W application. SimLab’s Statistical Pre-Processor module executes in the procedure (step 1, Fig.1), based on PDFs provided by the user and the method selected and produces a matrix of sample inputs to run the model (step 2, Fig. 1). A processor program was written in C# (C-sharp language) to automatically run VFSMOD once for each new set of sample inputs. The program automatically substitutes the new parameter set into the input files, runs the model, and performs the necessary post-processing tasks to obtain the selected model outputs for the analysis (step 3, Fig. 1). Outputs are stored in a matrix. The Statistical Post-Processor module of SimLab uses the input and output matrices to calculate the sensitivity indexes of the Morris and the Extended FAST methods (step 4, Fig. 1). The Data Analysis Toolpack of the Excel spreadsheet software (Microsoft Corp. Redmond, Washington, USA), was used to construct the output probability distributions and to quantify the uncertainty based on the set of Extended FAST results step 6, (Fig. 1).

2.3. Application Case

The vegetative filter strip (VFS) modeling design system, VFSMOD-W (Muñoz-Carpena et al., 1999; Muñoz-Carpena and Parsons, 2004; Muñoz-Carpena and Parsons, 2005) was used for this application. VFSMOD—W contains two components, the main program, VFSTM, and a front-end program, UH, selectable by the user through the MS-Windows graphical user interface (GUI). VFSTM is a field-scale, mechanistic, storm-based model developed to route the incoming hydrograph and sedigraph from an adjacent field through a VFS and to calculate the resulting outflow, infiltration, and sediment trapping efficiency. A hydrology sub-model in VFSTM routes the input overland flow from the source area through the filter by solving the kinematic wave equation using finite elements (Muñoz-Carpena et al., 1993a,b) coupled with the extended Green-Ampt equation to handle infiltration from natural storm hyetographs (Skaggs and Khaheel, 1982). The hydrology subroutine is linked to a sub-model for filtration of suspended solids by artificial grass media (Tollner et al., 1976, 1977) and later tested for field conditions (Barfield et al. 1978, 1979; Hayes et al., 1979, 1984; Wilson et al., 1981). When no measured VFS input data is available the UH front-end component can be selected to generate source area inputs for each design storm, including a rainfall hyetograph, a runoff hydrograph, and sediment loss from the source area using a combination of the NRCS curve number, the unit hydrograph, and the modified Universal Soil Loss Equation methods. With these inputs (Table 1), a set of response curves, i.e., sediment and runoff reduction vs. filter design/construction characteristics (filter length, width, grass type, slope), can be developed from VFSMOD—W outputs for a given design scenario (Muñoz-Carpena and Parsons, 2004). visit: http://carpena.ifas.ufl.edu/vfsmod/ for more information on VFSMOD-W.

Although analysis of sensitivity and uncertainty of the model have been previously reported (Muñoz-Carpena et al., 1999; Abu-Zreig, 2001; Parsons and Muñoz-Carpena, 2001; Shirmohammadi et al., 2006), only classical local OAT approaches were used. These studies serve as the basis for comparison with the global techniques presented here.

The specific conditions selected for evaluation of the model are those of the phosphate-mining region of central Florida along the Peace River basin (Fig. 2). Continued mining has degraded water quality in the Peace River watershed and has left large mounds of refuse material that now shape the landscape surrounding the river. The mound material is essentially homogenous clean sand (>94% in weight) with a high concentration in apathite, the P mineral ore, and mixed with small pockets of clay in some points.
Table 1. Simulation parameters for the combined VFSMOD-W model (source area UH and grass buffer VFSM components)

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter (source area simulation parameters)</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
<td>mm</td>
<td>Design Storm precipitation</td>
</tr>
<tr>
<td>2</td>
<td>CN</td>
<td>--</td>
<td>SCS curve number for source area</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>ha</td>
<td>Area of upstream portion</td>
</tr>
<tr>
<td>4</td>
<td>Storm type</td>
<td>--</td>
<td>NRSC Storm type (1=I, 2=II, 3=III, 4=Ia)</td>
</tr>
<tr>
<td>5</td>
<td>D</td>
<td>h</td>
<td>Storm duration</td>
</tr>
<tr>
<td>6</td>
<td>L</td>
<td>m</td>
<td>Length of the source area along the slope</td>
</tr>
<tr>
<td>7</td>
<td>Y</td>
<td>m/m</td>
<td>Slope of the source area [m/m]</td>
</tr>
<tr>
<td>8</td>
<td>Soil type</td>
<td>--</td>
<td>USDA texture for source area top soil (label)</td>
</tr>
<tr>
<td>9</td>
<td>K</td>
<td>(kg.h)/(m².N)</td>
<td>USLE soil erodibility index</td>
</tr>
<tr>
<td>10</td>
<td>C</td>
<td>--</td>
<td>USLE cover and management factor</td>
</tr>
<tr>
<td>11</td>
<td>Pfact</td>
<td>--</td>
<td>USLE conservation practice factor</td>
</tr>
<tr>
<td>VFSM (vegetative filter strip simulation parameters)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>FWIDTH</td>
<td>m</td>
<td>Effective flow width of the strip</td>
</tr>
<tr>
<td>13</td>
<td>VL</td>
<td>m</td>
<td>Length of the filter (flow direction)</td>
</tr>
<tr>
<td>14</td>
<td>RNA(I)</td>
<td>s/m¹³</td>
<td>Filter Manning’s roughness n for each segment</td>
</tr>
<tr>
<td>15</td>
<td>SOA(I)</td>
<td>m/m</td>
<td>Filter slope for each segment</td>
</tr>
<tr>
<td>16</td>
<td>VKS</td>
<td>m/s</td>
<td>Soil vertical saturated hydraulic conductivity in the VFS, Kₛ</td>
</tr>
<tr>
<td>17</td>
<td>SAV</td>
<td>m</td>
<td>Green-Ampt’s average suction at wetting front</td>
</tr>
<tr>
<td>18</td>
<td>OS</td>
<td>m³/m³</td>
<td>Saturated soil water content, θₛ</td>
</tr>
<tr>
<td>19</td>
<td>OI</td>
<td>m³/m³</td>
<td>Initial soil water content, θᵢ</td>
</tr>
<tr>
<td>20</td>
<td>SM</td>
<td>m</td>
<td>Maximum surface storage</td>
</tr>
<tr>
<td>21</td>
<td>SCHK</td>
<td>--</td>
<td>Relative distance from the upper filter edge where check for ponding conditions is made (i.e. 1 = end, 0.5 = mid point, 0 = beginning)</td>
</tr>
<tr>
<td>22</td>
<td>SS</td>
<td>cm</td>
<td>Average spacing of grass stems</td>
</tr>
<tr>
<td>23</td>
<td>VN</td>
<td>s/cm¹³</td>
<td>Filter media (grass) modified Manning’s n (0.012 for cylindrical media)</td>
</tr>
<tr>
<td>24</td>
<td>H</td>
<td>cm</td>
<td>Filter grass height</td>
</tr>
<tr>
<td>25</td>
<td>VN2</td>
<td>s/cm¹³</td>
<td>Bare surface Manning’s n for sediment inundated area in grass filter</td>
</tr>
<tr>
<td>26</td>
<td>DP</td>
<td>cm</td>
<td>Sediment particle size diameter (d₅₀)</td>
</tr>
<tr>
<td>27</td>
<td>COARSE</td>
<td>--</td>
<td>Fraction of incoming sediment with particle diameter &gt; 0.0037 cm (coarse fraction routed through wedge as bed load) [unit fraction, i.e. 100% = 1.0]</td>
</tr>
</tbody>
</table>

Currently, there is interest to study the potential of vegetative filter strips as a best management practice (BMP) to use in the mitigation plans that are required as part of the mining permitting process by the State of Florida. Field experiments are being conducted to quantify runoff quantity and quality from the refuse mining mounds and the effectiveness of VFS in the area (Kuo et al., 2005; Kuo, 2007). Values from these experiments are used as the basis for the global model evaluation described below.

2.4. Selection of Input PDF’s and Model Outputs

The input factors of the model used in the example analysis (Table 1) were assigned ranges and PDFs representative of the application area in Bartow, FL. Normal distributions were fitted to parameters with centrally distributed frequencies and a sufficient amount of data available (n>50), like source area and filter slopes (Y and SOA) and grass height (H) (Fig. 3). A log-normal PDF was selected for soil saturated hydraulic conductivity of the filter (VKS) and for the fraction of coarse sediment (COARSE) to match the uniform particle size distribution of the soil and sediment from the area. The beta (β) distribution was used for FWIDTH (effective flow width of the strip) and OI (initial soil water content) to match their smooth but biased (to the right of the mean) distributions (Fig. 3). Factors with reduced number of measurements like parameters related to vegetation (SS, VN), soil (CN, K, RNA, OS) and sediment particle diameter (DP) were given triangular PDFs based on the limited data available. Finally, the user-selectable parameter SCHK (node to check ponding during infiltration calculations) and Green-Ampt suction at the wetting front (SAV) were assigned uniform distributions since no known frequency pattern was identified through their ranges (Fig. 3). The rest of the parameters, such as, the optimal filter design lengths (VL), soil textural class (sand), and design storm characteristics (P, Storm type) selected for a return period of T=10 yr, were considered fixed since they were known for the area of application. Other factors like C, Pfact and SM were fixed to represent the worst case conditions. All input factors were assumed to be independent of each other.
Finally, in addition to the VFSMOD-W model parameters, the effect of the model structure (VFSM with or without UH component) was also considered as an input factor in the analysis.

Several model outputs were selected in the analysis to represent the potential variability of the hydrology and sediment transport components of the model (Muñoz-Carpena et al., 1999) (Table 2).

<table>
<thead>
<tr>
<th>Component</th>
<th>Output</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrology</td>
<td>UH</td>
<td>TRS (mm) (depth over source area)</td>
<td>Total runoff from source into filter</td>
</tr>
<tr>
<td></td>
<td>VFSM</td>
<td>TRF (mm) (depth over source+filter)</td>
<td>Total runoff output from filter</td>
</tr>
<tr>
<td></td>
<td>VFSM</td>
<td>TIF (mm) (over filter)</td>
<td>Total infiltration in filter</td>
</tr>
<tr>
<td></td>
<td>VFSM</td>
<td>RDR --</td>
<td>Runoff delivery ratio (flow out from filter / flow in)</td>
</tr>
<tr>
<td>Sediment</td>
<td>UH</td>
<td>MSS (kg)</td>
<td>Mass sediment Input from source area</td>
</tr>
<tr>
<td></td>
<td>UH</td>
<td>CSS (g/L)</td>
<td>Concentration sediment in runoff from source area</td>
</tr>
<tr>
<td></td>
<td>VFSM</td>
<td>MSF (kg)</td>
<td>Mass sediment output from filter</td>
</tr>
<tr>
<td></td>
<td>VFSM</td>
<td>CSF (g/L)</td>
<td>Concentration sediment in runoff exiting the filter</td>
</tr>
<tr>
<td></td>
<td>VFSM</td>
<td>MSR (kg)</td>
<td>Mass sediment retained in filter</td>
</tr>
<tr>
<td></td>
<td>VFSM</td>
<td>SDR --</td>
<td>Sediment delivery ratio (mass out from filter / mass in)</td>
</tr>
<tr>
<td></td>
<td>VFSM</td>
<td>EFL (m)</td>
<td>Effective filter length</td>
</tr>
<tr>
<td></td>
<td>VFSM</td>
<td>WD (m)</td>
<td>Sediment Wedge Distance</td>
</tr>
</tbody>
</table>

Two of these outputs Runoff Delivery Ratio and Sediment Delivery Ratio (RDR and SDR), have been proposed as the objective functions for the design of the VFS (Muñoz-Carpena and Parsons, 2004). Further sensitivity and uncertainty analyses discussed in this work, are mainly focused on those VFS performance outputs.

Four input sample sets were generated for the Morris and FAST methods. Each sample set represented a combination of the two model structures (VFSM or UH/VFSM) and one of design filter lengths studied in the area (VL=3m or 6m). The number of model runs for each method was selected according to the number of uncertain parameters in each model structure based on equations (2) and (6). The number of simulations run for each method (Table 3) illustrates one of the potential advantages of the Morris method over FAST, i.e. the relatively shorter computation time needed.

<table>
<thead>
<tr>
<th>Model structure</th>
<th>VL (m)</th>
<th>Morris</th>
<th>FAST</th>
<th>Total runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>VFSM</td>
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<td>14</td>
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<td>4</td>
</tr>
<tr>
<td>UH/VFSM</td>
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<td>16</td>
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</tr>
<tr>
<td>VFSM</td>
<td>6</td>
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<tr>
<td>UH/VFSM</td>
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<tr>
<td>Total sims.</td>
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</table>

3.- Results and discussion

3.1. Global Sensitivity Analysis

3.1.1. Screening Method: The Method of Morris

Fig. 4 shows the graphical representation of the Morris values for a selected subset of outputs for filter design length of 6 m (VL=6m). As suggested by Morris, the ranking of importance of the input factors can be based on the relative value of $\mu^*$. Furthermore, only parameters separated from the origin of the $\mu^*\sigma$ plane are considered important.

The number of important parameters identified was effectively smaller than the full set of model inputs studied (down to 4 from the original 14 for VFSM and to 6 from 16 for UH/VFSM).

The previous study on local OAT sensitivity of VFSM (Muñoz-Carpena et al., 1999) indicated that the main parameters controlling the runoff from the filter were VKS and OI. However, no relative ranking of the inputs could be produced. The Morris analysis of VFSM shows a strong influence of VKS on RDR and TRF, although OI is not found important relative to them. This is shown in Fig. 4 for the RDR output (top of the figure) where only VKS is shown away from the $\mu^*\sigma$ plane. Since the important parameters is closer to the $\mu^*$ axis its influence is mostly through first order effects with a small interaction component. The infiltration in the filter (TIF) is influenced mainly by VKS. As expected, VKS, the Green-Ampt infiltration parameter, controls the infiltration in the VFS.
Outputs from VFSM alone related to sediment outflow from the filter (MSF and SDR) are controlled in order of importance mainly by DP and FWIDTH with smaller influence of VKS. In case of the filter outflow sediment concentration (CSF), DP appears as the most important of the three. This can be explained by the fact that the sediment concentration is calculated directly from both mass of sediment and water outflow, where MSF and SDR represent dry mass of sediment alone. Previous local OAT testing of the sediment component of VFSM showed that the main parameters controlling sediment outflow were particle class and the media spacing (Muñoz-Carpena et al., 1999). The high ranking of DP is in agreement with findings of Abu-Zreig (2001) that sediment class is influential for SDR. Lack of influence by SS observed in current results could be explained by the relatively small range of this factor, measured for the Bartow conditions. Abu-Zreig (2001) also identified FWIDTH as an important factor for the VFS performance (Abu-Zreig et al. 2001). This is confirmed by the Morris analysis that identifies FWIDTH as the second or third most important parameter for the filter sediment outputs for all combinations studied.

Sediment related outputs for the UH/VFSM structure show an importance of the source area erosion parameters in UH (Y, K and CN), additionally to those controlling sediment dynamics in the VFSM component alone (DP and FWIDTH). In some cases it is difficult to differentiate the relative importance of some parameters like VKS, CN, K and DP in CSF or DP and CN, K and FWIDTH in SDR for UH/VFSMOD (Fig. 4). This can introduce an element of subjectivity in the Morris analysis. As seen in Fig. 4, for all important parameters and sediment outputs, the interactions seemed limited.

These initial screening results clearly illustrate four of the products of the global sensitivity analysis: importance of the parameters for different outputs, effect of changing modeling structure, assurance on the model’s behavior (absence of errors), type of influence of the important parameters (first order or interactions).

3.1.2. Variance-Based Method: Extended FAST

The subset of important parameters selected by the screening method for each model combination was used for further analysis with the extended FAST method. These parameters were: VKS, DP and for VFSM model structure, and additionally Y, K, and CN for UH/VFSM model structure. The results for selected outputs and the filter length of VL = 6m, are presented in Fig. 5. This figure depicts the fraction of the total output variance explained by each parameter (vertical axis) for each of the selected outputs (horizontal axis). The first order linear effects ($S_i$) are presented first for each model structure (Fig. 5 a,c) followed by the higher order interactions ($S_{i-ST_i}$) (Fig. 5 b,d).

The Extended FAST results obtained reinforce and quantify those from Morris method, and in some cases eliminate the subjectivity introduced in the qualitative approach of the later (Fig. 5a). For the CSF output it is know easy to separate VKS, CN, K and DP (with 13%, 10%, 8% and 9% variance explained). Similarly, the variance of SDR can be attributed in 30% to DP, 25% to CN, 5% and 6% to K and FWIDTH, respectively.

The Extended FAST results are considered more reliable than Morris results, since they are based on a much larger number of simulations (Table 3) and less structured sampling scheme.

The sum of the total effects ($\Sigma S_i$) is graphically presented for both model structures by a thick line in Fig. 5a and 5c. In general, the sum of first order effect is greater than 80% of the total variance for most outputs for both model structures, with the exception of the sediment wedge dimensions EFL and WD for UH/VSMOD (50% of total variance). The general additivity of the model can lead to an efficient calibration in most field situations if reliable data is provided.
3.1.3. Global Uncertainty Analysis from Extended FAST Results

Following Morgan and Henrion (1992, Chapter 9), to communicate the uncertainty graphically to end-users the density and cumulative probability distributions (CDFs) were constructed for the selected outputs and both model structures (Fig. 6). Another method recommended by these authors, the 95% confidence interval (i.e. range of output values between 2.5% and 92.5% cumulative distribution percentiles) can also be calculated on the basis of these distributions.

Example uncertainty analysis statistics for the Sediment Delivery Ratio obtained from FAST results are presented in Fig. 6.

The difference between CDFs for VFSM and UH/VFSM illustrates the relative effects of model structure on output uncertainties. Generally larger output variances are observed for UH/VFSM, which is expected since additional variance is introduced by the larger number of uncertain parameters. The filter length does not systematically affect the ranges of the PDFs.

The uncertainty of the results can also be communicated as probability of exceedance of a desired regulatory or design value. For example if 75% reduction of runoff sediment (SDR≤0.25) by the VFS is desired for the 10-yr design storm in the area, the results in Fig. 6 indicate that for the 6 m filter this probability will be approximately 5% for the combined model and approximately 7% for simpler model structure. The 6 m filter will perform as desired in, respectively, 95% and 93% of events. In the case of the 3 m filter, the probability of exceedance of a desired SDR=0.25 will be greater that 75% percent of the cases for two model structures, which indicates that this filter does not meet the design criteria in the area of application.

4.- Summary and Conclusions

A model evaluation framework was proposed for hydrological and water quality models combining two types of global sensitivity analysis techniques (screening method of Morris and variance-based Extended FAST) and uncertainty analysis (based on Extended FAST results).

The use of the proposed framework is illustrated through an application to evaluate the vegetative filter strip modeling system VFSMOD-W and comparison with a previous OAT sensitivity analysis of the model.

The results illustrate four possible products of global sensitivity analysis: ranking of importance of the parameters for different outputs, effect of changing modeling structure, assurance on the model’s behavior, type of influence of the important parameters (first order or interactions). The proposed framework provided further validation of the model quality since no errors were detected regarding the model behavior (all the relations between...
inputs and outputs could be explained on the basis of the model assumptions. In contrast to previously performed OAT sensitivity analysis of the model, the method of Morris was able to provide a ranking of the significant parameters for a variety of outputs. In the case of the runoff and sediment delivery ratios (RDR and SDR), used as objective functions in the filter design, for the simpler model structure (VFSM component alone), VKS was identified as the controlling parameter in respect to the RDR whereas the order of importance with respect of SDR was DP > FWIDTH> VKS. Additionally, for the combined UH/VFSM model structure the source area parameters: Y, CN, K for SDR were ranked above VFSM parameters: FWIDTH and VKS (but not DP). FAST results reinforced and quantified those of Morris and indicated the additive nature of the model (sum of first order effects, $S_i>0.8$ for RDR and SDR) that can lead to its effective calibration if reliable input data is available. As expected, the predicted model uncertainty was higher for UH/VFSM than for VFSM since more uncertain inputs were used in the combined model. The evaluated uncertainties of the model outputs also varied depending on the filter length. Based on the uncertainty the probability of exceedance of a desired SDR=0.25 (filter traps 75% of input sediment) was found to be approximately 5% and 7% for UH/VSMOD and VFSM model structures (acceptable at 95% and 90% uncertainty level) for the 6 m filter, while for the alternative 3 m filter this was over 75% (unacceptable at 90% uncertainty level).

No evaluation method can be considered objective since it relies on the interpretation by the modeller of the input variation. Yet, the proposed global analysis framework is found robust and reproducible since it considers concurrent variation of the input factors without a priori judgment of their relative importance over the output.

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